

Cosmological implications of interacting polytropic gas dark energy model in non-flat universe

M. Malekjani ^{1,2*}, A. Khodam-Mohammadi ^{1†} and M. Taji¹

¹*Department of Physics, Faculty of Science,*

Bu-Ali Sina University, Hamedan 65178, Iran

²*Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran*

Abstract

The polytropic gas model is investigated as an interacting dark energy scenario. The cosmological implications of the model including the evolution of EoS parameter w_Λ , energy density Ω_Λ and deceleration parameter q are investigated. We show that, depending on the parameter of model, the interacting polytropic gas can behave as a quintessence or phantom dark energy. In this model, the phantom divide is crossed from below to up. The evolution of q in the context of polytropic gas dark energy model represents the decelerated phase at the early time and accelerated phase later. The singularity of this model is also discussed. Eventually, we establish the correspondence between interacting polytropic gas model with tachyon, K-essence and dilaton scalar fields. The potential and the dynamics of these scalar field models are reconstructed according to the evolution of interacting polytropic gas.

* Email: malekjani@basu.ac.ir

† Email: khodam@basu.ac.ir

I. INTRODUCTION

Recent cosmological observations obtained by SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4] experiments reveal that our universe expands under an accelerated expansion. In the framework of standard Friedmann-Robertson-Walker (FRW) cosmology, a missing energy component with negative pressure dubbed dark energy (DE) is responsible for this expansion. The nature of DE is still unknown and scientists believe that the problem of DE is a major puzzle of modern cosmology. Up to now, many theoretical models have been investigated to interpret the behavior of DE. The time-independent cosmological constant, Λ , with EoS parameter $w = -1$ is the earliest and simplest candidate of DE. The cosmological constant suffers from two well known difficulties namely "fine-tuning" and "cosmic coincidence" problems. The alternative candidates for DE problem are the dynamical dark energy scenario with time varying EoS parameter, w . According to some analysis on the SNe Ia observational data, it has been shown that the time-varying DE models give a better fit compare with a cosmological constant [5]. There are two different categories for dynamical DE scenario: (i) The scalar fields including quintessence [6], phantom [7], quintom [8], K-essence [9], tachyon [10], dilaton [11] and so forth. (ii) The interacting DE models including Chaplygin gas models [12, 13], braneworld models [14], holographic [15] and agegraphic [16] models. The holographic DE model is constructed in the light of holographic principle of quantum gravity [17] and the agegraphic model is constructed based on the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity [18]. The interaction between DE and dark matter is supported by recent observations prepared by the Abell Cluster A586 [19]. However the strength of this interaction is not clearly identified [20]. Also, recent astronomical data supported that our universe is not a perfectly flat and has a small positive curvature [21].

The polytropic gas model has an important role in stellar astrophysics. It can explain the equation of state of degenerate electrons and degenerate neutrons in white dwarfs and neutron stars, respectively [28]. This model can also be useful when the pressure and density are adiabatically related to each other in main sequence stars [28]. Here we consider the interpretation of dark energy scenario with the EoS parameter of polytropic gas. U. Mukhopadhyay and S. Ray used some dynamical Λ model with polytropic equation of state in dark energy scenario [22]. Recently, by using the polytropic gas model, the interaction

between DE and dark matter is investigated [24]. Karami, et al. obtained the phantom behavior of interacting polytropic gas model [24]. Also, karami, et al. reconstructed the $f(T)$ -gravity from the polytropic gas DE model [27]. They also studied the correspondence between the interacting new agegraphic dark energy model with polytropic gas model in non-flat FRW universe and reconstructed the potential and the dynamics for the scalar field of the polytropic model to describe the accelerated expansion of the universe [23]. The above statements motivate us to consider more cosmological implications of this model in dark energy scenario. One of the interesting features of this model that we discuss is that in the polytropic gas dark energy scenario the phantom regime can be achieved even in the absence of interaction between dark energy and dark matter. This makes it distinguishable from many other DE model whose W_Λ can not cross the phantom regime without the interaction between DE and dark matter. We consider the interacting polytropic gas as a phenomenological DE model. In the phenomenological models of DE the pressure p is given as a function of energy density ρ , i.e., $p = -\rho - f(\rho)$ [25]. Considering $f(\rho) = 0$, the EoS parameter of phenomenological models cross $w = -1$, i.e., the EoS of cosmological constant. Nojiri, et al. investigated four types singularities for some illustrative examples of phenomenological models [25]. The polytropic gas model has a type III. singularity in which the singularity takes place at a characteristic scale factor a_s .

Here, we obtain the deceleration parameter q to explain the decelerated and accelerated expansion phases of the universe dominated by polytropic gas dark energy fluid. The behavior of interacting polytropic gas in the quintessence regime is also calculated. We study the correspondence between the tachyon, K-essence and dilaton fields with the interacting polytropic gas dark energy and reconstruct the potential and the dynamics of these scalar fields according the evolutionary form of interacting polytropic gas model.

II. POLYTROPIC GAS DE MODEL

The equation of state (EoS) of polytropic gas is given by

$$p_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}}, \quad (1)$$

where K and n are the polytropic constant and polytropic index, respectively [28].

Assuming a non-flat Friedmann-Robertson-Walker (FRW) universe containing DE and CDM

components, the corresponding Friedmann equation is as follows

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_\Lambda) \quad (2)$$

where H is the Hubble parameter, M_p is the reduced Plank mass and $k = 1, 0, -1$ is a curvature parameter corresponding to a closed, flat and open universe, respectively. ρ_m and ρ_Λ are the energy density of CDM and DE, respectively. Recent observations support a closed universe with a tiny positive small curvature $\Omega_k \simeq 0.02$ [29].

The dimensionless energy densities are defined as

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{\rho_m}{3M_p^2 H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\rho_\Lambda}{3M_p^2 H^2}, \quad \Omega_k = \frac{k}{a^2 H^2} \quad (3)$$

Therefore the Friedmann equation (2) can be written as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \quad (4)$$

Considering a universe dominated by interacting polytropic gas DE and CDM, the total energy density, $\rho = \rho_m + \rho_\Lambda$, satisfies a conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (5)$$

However, by considering the interaction between DE and dark matter, the energy density of DE and dark matter does not conserve separately and in this case the conservation equations are given by

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (6)$$

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q, \quad (7)$$

where Q indicates the interaction between DE and CDM. Three forms of Q which have been extensively used in the literatures are [30]

$$Q = 3\alpha H\rho_\Lambda, \quad 3\beta H\rho_m, \quad 3\gamma H(\rho_\Lambda + \rho_m), \quad (8)$$

where α , β and γ are the dimensionless constants. The Hubble parameter H in the Q -terms is considered for mathematical simplicity. Indeed, the interaction forms in Eq.(8) are given by hand, since the Q in Eqs.(6, 7) should be as a function of H multiplied with energy density. Similar to the standard Λ CDM model, in which the vacuum fluctuations can decay

into matter, here the interaction parameter Q indicates the decay rate of the polytropic gas into CDM component. Recently, the interaction between DE and dark matter is presented in [31]. For mathematical simplicity, we consider the first form of interaction parameter Q .

Using Eq.(1), the integration of continuity equation for interacting dark energy component, i.e. Eq.(7), obtains

$$\rho_\Lambda = \left(\frac{1}{Ba^{\frac{3(1+\alpha)}{n}} - \tilde{K}} \right)^n, \quad (9)$$

where B is the integration constant, $\tilde{K} = \frac{K}{1+\alpha}$ and a is the scale factor. Note that to have a positive energy density for an arbitrary number of n , it is required $Ba^{3(1+\alpha)/n} > \tilde{K}$. It is worthwhile to note that the phantom behavior of interacting polytropic gas has been also studied in [24]. In the case of $Ba^{3(1+\alpha)/n} = \tilde{K}$, we have $\rho \rightarrow \infty$ and therefore the polytropic gas has a finite-time singularity at $a_c = (\tilde{K}/B)^{n/3(1+\alpha)}$. This type of singularity, in which at a characteristic scale factor a_s , the energy density $\rho \rightarrow \infty$ and the pressure density $|p| \rightarrow \infty$, is indicated by type III singularity [25].

Substituting $Q = 3\alpha H \rho_\Lambda$ in (7), we have

$$\dot{\rho}_\Lambda + 3H(1 + \alpha + w_\Lambda)\rho_\Lambda = 0, \quad (10)$$

Taking the derivative of Eq.(9) with respect to time, one can obtain

$$\dot{\rho}_\Lambda = -3BH(1 + \alpha)a^{\frac{3(1+\alpha)}{n}}\rho_\Lambda^{1+\frac{1}{n}} \quad (11)$$

Substituting Eq.(11) in (10) and using Eq.(9), we can obtain the EoS parameter of interacting polytropic gas as

$$w_\Lambda = -1 - \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha \quad (12)$$

where $c = \tilde{K}/B$. By defining the effective EoS parameter as $w_\Lambda^{eff} = w_\Lambda + \alpha = -1 - a^{\frac{3(1+\alpha)}{n}}/(c - a^{\frac{3(1+\alpha)}{n}})$, we see that the interacting polytropic gas model behaves as a phantom model, i.e. $w_\Lambda^{eff} < -1$, when $c > a^{3(1+\alpha)/n}$. The phantom behavior of polytropic gas is similar to generalized chaplygin gas model, where it has been shown that the generalized chaplygin gas with negative value of model parameter can behave as a phantom dark energy [13]. Note that in the case of phantom polytropic gas, from Eq.(9) we see that only even numbers of n should be chosen to have a positive energy density. The interesting feature of polytropic gas model is that it can obtain the phantom regime even in the

absence of interaction. For this aim, it is enough to insert $\alpha = 0$ in Eq.(12) and see that for $c > a^{\frac{3(1+\alpha)}{n}}$ the phantom regime, $w_\Lambda < -1$, can be achieved. This makes it distinguishable from many other dark energy models whose w_Λ cannot cross the phantom regime without interaction term. The other interesting aspect of the polytropic gas is that the interacting polytropic gas dark energy crosses the phantom divide from $w_\Lambda < -1$ to $w_\Lambda > -1$ (see Fig.(1), left panels). This behavior of polytropic gas is similar to interacting agegraphic dark energy model in which the phantom divide is crossed from below to up (see Figs.(2,3) of [26]). The similarity of the interacting agegraphic dark energy and polytropic gas is that both models cross the phantom divide from below to up.

The interacting polytropic gas behaves as a quintessence model, i.e. $-1 < w_\Lambda^{eff} < -1/3$, when $-\infty < c \leq -a^{3(1+\alpha)/n}/2$. The condition $-a^{3(1+\alpha)/n}/2 < c < a^{3(1+\alpha)/n}$ leads to $w_\Lambda^{eff} > -1/3$ and consequently the accelerated expansion, in this case, can not be achieved. At $c = a^{3(1+\alpha)/n}$, the interacting polytropic gas has a singularity. Hence, depending on the parameter c , the polytropic gas can behaves as a phantom or quintessence models of DE. Also it is worth to mention that the polytropic gas model behaves as a cosmological constant, i.e., $w_\Lambda^{eff} \rightarrow -1$, at the early time (i.e. $a \rightarrow 0$) whereas the universe is dominated by pressureless dark matter.

In Fig. (1), the evolution of w_Λ as a function of scale factor is plotted for different values of the parameters c and n . The other interesting aspect of the polytropic gas is that the interacting polytropic gas dark energy crosses the phantom divide from $w_\Lambda < -1$ to $w_\Lambda > -1$ (see Fig.(1), left panels). This behavior of polytropic gas is similar to interacting agegraphic dark energy model in which the phantom divide is crossed from below to up (see Figs.(2,3) of [26]). The similarity of the interacting agegraphic dark energy and polytropic gas is that both models cross the phantom divide from below to up. In upper panels we fix the polytropic index as $n = 2$ and in lower panels the parameter c is fixed. In upper left panel the negative values of c are selected to obtain the transition from phantom to quintessence regime. In upper right panel, the positive values of c are selected. In this case the interacting polytropic gas behaves as a phantom like field. Same as left panel, we fix the polytropic index $n = 2$. Here, one can easily find the phantom behavior of polytropic gas model. It is worth noting that the phantom regime of polytropic gas model is restricted with a characteristic scale factor $a_s = c^{n/3(1+\alpha)}$, where we encounter with a singularity at this epoch. In lower panels of Fig.(1), the dependency of the evolution of w_Λ on the

polytropic index parameter n is studied. In lower left panel, by fixing $c = -1$, we studied this dependency for polytropic gas model. It is easy to see that the larger value of n gets the larger w_Λ at $a < 1$ and smaller w_Λ at $a > 1$. In lower right panel, by fixing $c = 2$, the dependency of w_Λ on the parameter n is investigated for phantom polytropic gas model. Unlike to lower left panel, the larger value of n gets the smaller w_Λ at $a < 1$ and larger w_Λ at $a > 1$.

In order to obtain the evolution of dimensionless energy density, Ω_Λ , let us start with Eqs.(9) and (3) and obtain the density parameter of interacting polytropic gas as

$$\Omega_\Lambda = \frac{(Ba^{\frac{3(1+\alpha)}{n}} - \tilde{K})^{-n}}{3M_p^2 H^2} \quad (13)$$

Taking the derivative of Eq.(13) with respect to time and using $\Omega' = \dot{\Omega}/H$, we can obtain

$$\Omega'_\Lambda = -\Omega_\Lambda \left(\frac{3(1+\alpha)a^{\frac{3(1+\alpha)}{n}}}{a^{\frac{3(1+\alpha)}{n}} - c} + 2\frac{\dot{H}}{H^2} \right) \quad (14)$$

where prime denotes the derivative with respect to $x = \ln a$. Taking the derivative of Friedmann equation (2) with respect to time and using Eqs.(9), (4), (6), (13) and $Q = 3\alpha H \rho_\Lambda$, one can find that

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \left[\Omega_\Lambda \frac{c(1+\alpha)}{a^{\frac{3(1+\alpha)}{n}} - c} + 1 + \frac{\Omega_k}{3} \right] \quad (15)$$

Substituting this relation into Eq.(14), we obtain the evolutionary equation for energy density parameter of interacting polytropic gas as:

$$\Omega'_\Lambda = -3\Omega_\Lambda \left[\frac{c}{a^{\frac{3(1+\alpha)}{n}} - c} (1 - \Omega_\Lambda) + a \frac{a^{\frac{3(1+\alpha)}{n}} - c\Omega_\Lambda}{a^{\frac{3(1+\alpha)}{n}} - c} - \frac{\Omega_k}{3} \right], \quad (16)$$

where Ω_k is given by

$$\Omega_k = a\gamma \frac{1 - \Omega_\Lambda}{1 - a\gamma} \quad (17)$$

and $\gamma = \Omega_{k0}/\Omega_{m0}$.

In Fig.(2), by solving the differential equation (16), we show the evolution of Ω_Λ for different model parameters c and n as well as different interaction parameter α . Here we assume only the positive values of c , i.e., the phantom polytropic gas model. The numerical values of density parameters at the present time are taken as: $\Omega_{\Lambda 0} = 0.7$, $\Omega_{m0} = 0.3$ and $\Omega_{k0} = 0.02$. In upper panels, we consider the non-interacting polytropic gas and in lower panel the

interaction term is included. Here, we see that $\Omega_\Lambda \rightarrow 0$ at the early time and tends to 1 at the late time. Hence the polytropic gas model can describe the matter-dominated universe in the far past. Also, at the late time, we encounter with dark energy dominated universe ($\Omega_\Lambda \rightarrow 1$). In upper left panel, by fixing the parameter n , the polytropic gas starts to be effective earlier and Ω_Λ tends to a lower value at the late time when c is larger. On the other hand, in upper right panel, we see that for fixed parameter c , the polytropic gas starts to be effective earlier and also Ω_Λ tends to a higher value at the late time when n is smaller. In lower panel, the effect of interaction parameter α on the evolution of Ω_Λ is studied. Here one can see that the polytropic gas starts to be effective earlier, by increasing the interaction parameter α . Also, at $a > 1$, the parameter Ω_Λ is smaller for larger values of α .

For completeness, we derive the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2} \quad (18)$$

for polytropic gas model. Substituting Eq.(15) in (18) we get

$$q = -1 + \frac{3}{2} \left[\Omega_\Lambda \frac{c(1+\alpha)}{a^{\frac{3(1+\alpha)}{n}} - c} + 1 + \frac{\Omega_k}{3} \right] \quad (19)$$

It is worth noting that in the limiting case of matter-dominated phase and considering flat universe in the absence of interaction term, Eq.(19) is reduced to $q = 1/2$ which represents the decelerated expansion ($q > 0$) of the universe.

In Fig.(3), we show the evolution of q as a function of a for different model parameters c and n as well as different interaction parameter α . Here we discuss the evolution of q for phantom polytropic gas model, by assuming positive c . Upper panels is plotted in the absence of interaction between dark energy and dark matter and the lower panel is plotted in the presence of interaction term. The parameter q converges to $1/2$ at the early time, whereas the universe is dominated by pressureless dark matter. In upper left panel, by fixing n , the accelerated expansion is achieved earlier by increasing c . Also, in the upper right panel, we see that by increasing n , q becomes larger at the deceleration phase and gets smaller at the acceleration phase. It is worth noting that, although, both the model parameters n and c impact the evolution of deceleration parameter q , but the change of sign from $q > 0$ to $q < 0$ depends on the parameter c of the model.

III. CORRESPONDENCE BETWEEN POLYTROPIC GAS DE MODEL AND SCALAR FIELDS

In the present section we establish a correspondence between the interacting polytropic gas model with the tachyon, K-essence and dilaton scalar field models. The importance of this correspondence is that the scalar field models are an effective description of an underlying theory of dark energy and therefore it is worthwhile to reconstruct the potential and the dynamics of scalar fields according the evolutionary form of polytropic gas model. For this aim, first we compare the energy density of polytropic gas model (i.e. Eq.9) with the energy density of corresponding scalar field model. Then, we equate the equations of state of scalar field models with the EoS parameter of polytropic gas (i.e. Eq.12).

A. Polytropic gas tachyon model

It is believed that the tachyon can be assumed as a source of DE [33]. The tachyon is an unstable field which can be used in string theory through its role in the Dirac-Born-Infeld (DBI) action to describe the D-bran action [34]. The effective Lagrangian for the tachyon field is given by

$$\mathcal{L} = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi},$$

where $V(\phi)$ is the potential of tachyon. The energy density and pressure of tachyon field are [34]

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad (20)$$

$$p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \quad (21)$$

The EoS parameter of tachyon can be obtained as

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1. \quad (22)$$

In order to have a real energy density for tachyon field, it is required that $-1 < \dot{\phi} < 1$. Consequently, from Eq.(22), the EoS parameter of tachyon is constrained to $-1 < w_\phi < 0$. Hence, the tachyon field can interpret the accelerates expansion of universe, but it can not enter the phantom regime, i.e. $w_\Lambda < -1$. In order to reconstruct the potential and the

dynamics of tachyon according to evolution of interacting polytropic gas model, we should equate Eqs.(12) and (22) and also Eq.(9) with Eq.(20) as follows

$$w_\Lambda = -1 - \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha = \dot{\phi}^2 - 1. \quad (23)$$

$$\rho_\Lambda = \left(\frac{1}{Ba^{\frac{3(1+\alpha)}{n}} - \tilde{K}} \right)^n = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (24)$$

Hence we get the following expressions for dynamics and potential of tachyon field

$$\dot{\phi}^2 = -\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha \quad (25)$$

$$V(\phi) = \sqrt{1 + \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + \alpha} \left(\frac{1}{Ba^{\frac{3(1+\alpha)}{n}} - \tilde{K}} \right)^n \quad (26)$$

For $c > a^{3(1+\alpha)/n}$, from Eq.(25), we obtain $\dot{\phi}^2 < 0$ which represents the phantom behavior of tachyon field. It is worth noting that the reconstructed tachyon field according to the interacting polytropic gas can cross the phantom divide. By definition $\phi = i\psi$ and changing the time derivative to the derivative with respect to logarithmic scale factor, i.e. $d/dt = Hd/dx$, the scalar field ψ can be integrated from Eq.(25) as follows

$$\psi(x) - \psi(0) = \int_0^x \frac{1}{H} \sqrt{-\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha} dx \quad (27)$$

B. Polytropic gas K-essence model

The idea of the K-essence scalar field was motivated from the Born-Infeld action of string theory and can explain the late time acceleration of the universe [35]. The general scalar field action for K-essence model as a function of ϕ and $\chi = \dot{\phi}^2/2$ is given by [36]

$$S = \int d^4x \sqrt{-g} p(\phi, \chi), \quad (28)$$

where the Lagrangian density $p(\phi, \chi)$ relates to a pressure density and energy density through the following equations:

$$p(\phi, \chi) = f(\phi)(-\chi + \chi^2), \quad (29)$$

$$\rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2). \quad (30)$$

Hence, the EoS parameter of K-essence scalar field is obtained as

$$\omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}. \quad (31)$$

By comparing Eqs.(12) and (31), we have

$$w_\Lambda = -1 - \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha = \frac{\chi - 1}{3\chi - 1} \quad (32)$$

Hence the parameter χ is obtained as

$$\chi = \frac{2 + \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + \alpha}{4 + 3\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + 3\alpha} \quad (33)$$

From Eq.(31), one can see the phantom behavior of K-essence scalar field ($w_K < -1$) when the parameter χ lies in the interval $1/3 < \chi < 1/2$.

Using $\dot{\phi}^2 = 2\chi$ and changing the time derivative to the derivative with respect to $x = \ln a$, we obtain

$$\phi' = \frac{1}{H} \sqrt{\frac{4 + 2\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + 2\alpha}{4 + 3\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + 3\alpha}} \quad (34)$$

The integration of Eq.(34) yields

$$\phi(x) - \phi(0) = \int_0^x \frac{1}{H} \sqrt{\frac{4 + 2\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + 2\alpha}{4 + 3\frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} + 3\alpha}} dx \quad (35)$$

Here, we reconstructed the potential and the dynamics of K-essence scalar field according to the evolutionary form of the interacting polytropic gas model. The K-essence polytropic gas model can explain the accelerating universe and also behaves as a phantom model provided $1/3 < \chi < 1/2$.

C. Polytropic gas dilaton model

A dilaton scalar field can also be assumed as a source of DE. This scalar field is originated from the lower-energy limit of string theory [37]. The dilaton field is described by the effective Lagrangian density as

$$p_D = -\chi + ce^{\lambda\phi}\chi^2, \quad (36)$$

where c and λ are positive constant. Considering the dilaton field as a source of the energy-momentum tensor in Einstein equations, one can find that the Lagrangian density corresponds to the pressure of the scalar field and the energy density of dilaton field is also obtained as

$$\rho_D = -\chi + 3ce^{\lambda\phi}\chi^2, \quad (37)$$

Here $2\chi = \dot{\phi}^2$. The negative coefficient of the kinematic term of the dilaton field in Einstein frame makes a phantom like behavior for dilaton field. The EoS parameter of dilaton is given by

$$\omega_D = \frac{p_D}{\rho_D} = \frac{-1 + ce^{\lambda\phi}\chi}{-1 + 3ce^{\lambda\phi}\chi}. \quad (38)$$

In order to consider the dilaton field as a description of polytropic gas, we establish the correspondence between the dilaton EoS parameter, w_D , and the EoS parameter w_Λ of polytropic gas model. By equating Eq.(38) with Eq.(12), we find

$$ce^{\lambda\phi}\chi = \frac{w_\Lambda - 1}{3w_\Lambda - 1} = \frac{2 + \frac{a \frac{3(1+\alpha)}{n}}{c-a \frac{3(1+\alpha)}{n}} + \alpha}{4 + 3 \frac{a \frac{3(1+\alpha)}{n}}{c-a \frac{3(1+\alpha)}{n}} + 3\alpha} \quad (39)$$

By using $\chi = \dot{\phi}^2/2$ and $\dot{\phi} = \phi'H$, the scalar field ϕ can be obtained as

$$\phi(x) = \frac{2}{\lambda} \ln \left(e^{\lambda\phi(0)/2} + \frac{\lambda}{\sqrt{2c}} \int_0^x \frac{1}{H} \sqrt{\frac{2 + \frac{a \frac{3(1+\alpha)}{n}}{c-a \frac{3(1+\alpha)}{n}} + \alpha}{4 + 3 \frac{a \frac{3(1+\alpha)}{n}}{c-a \frac{3(1+\alpha)}{n}} + 3\alpha}} dx \right) \quad (40)$$

Here we presented the reconstructed potential and dynamics of dilaton scalar field according to the evolution of interacting polytropic gas model.

IV. CONCLUSION

In this work we presented the interacting polytropic gas model of dark energy to interpret the accelerated expansion of the universe. Assuming a non-flat FRW universe dominated by interacting polytropic gas DE and CDM, we studied the cosmic behavior of polytropic gas model. For this aim, we calculated the evolution of effective EoS parameter and showed that for positive values $c > a^{3(1+\alpha)/n}$ with even numbers of n this model behaves as a phantom DE model and in the case of $-\infty < c \leq -a^{3/n}/2$ it treats as a quintessence

model. Similar to interacting agegraphic dark energy model, the interacting polytropic gas model crosses the phantom divide from below ($w_\Lambda < -1$) to up ($w_\Lambda > -1$). The transition from phantom to quintessence depends on the parameter c of the model. For larger value of c , the transition take place sooner. In the case of phantom polytropic gas model, w_Λ is larger for larger value of c . In the scenario of polytropic gas model, the phantom divide can be crossed even in the absence of interaction. We also calculated the evolution of energy density Ω_Λ . The matter dominated phase at the early time and DE-dominated universe at the late time can be described in the context of polytropic gas model. The polytropic gas starts to be effective earlier for larger value of interaction parameter as well as for larger value of the parameter c or smaller value of n . We calculated the deceleration parameter q and obtained the decelerated and accelerated expansion phases of the universe in the context of polytropic gas model. The transition from decelerated expansion ($q > 0$) to accelerated expansion ($q < 0$) takes place sooner for larger value of c and also by increasing the interaction parameter α . Since the scalar fields models are the underlying theory of dark energy, we proposed a correspondence between interacting polytropic gas model with the tachyon, K-essence and dilaton scalar fields models. We reconstructed the potential and the dynamics of these scalar fields according to the evolution of the interacting polytropic gas model.

Acknowledgments

This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

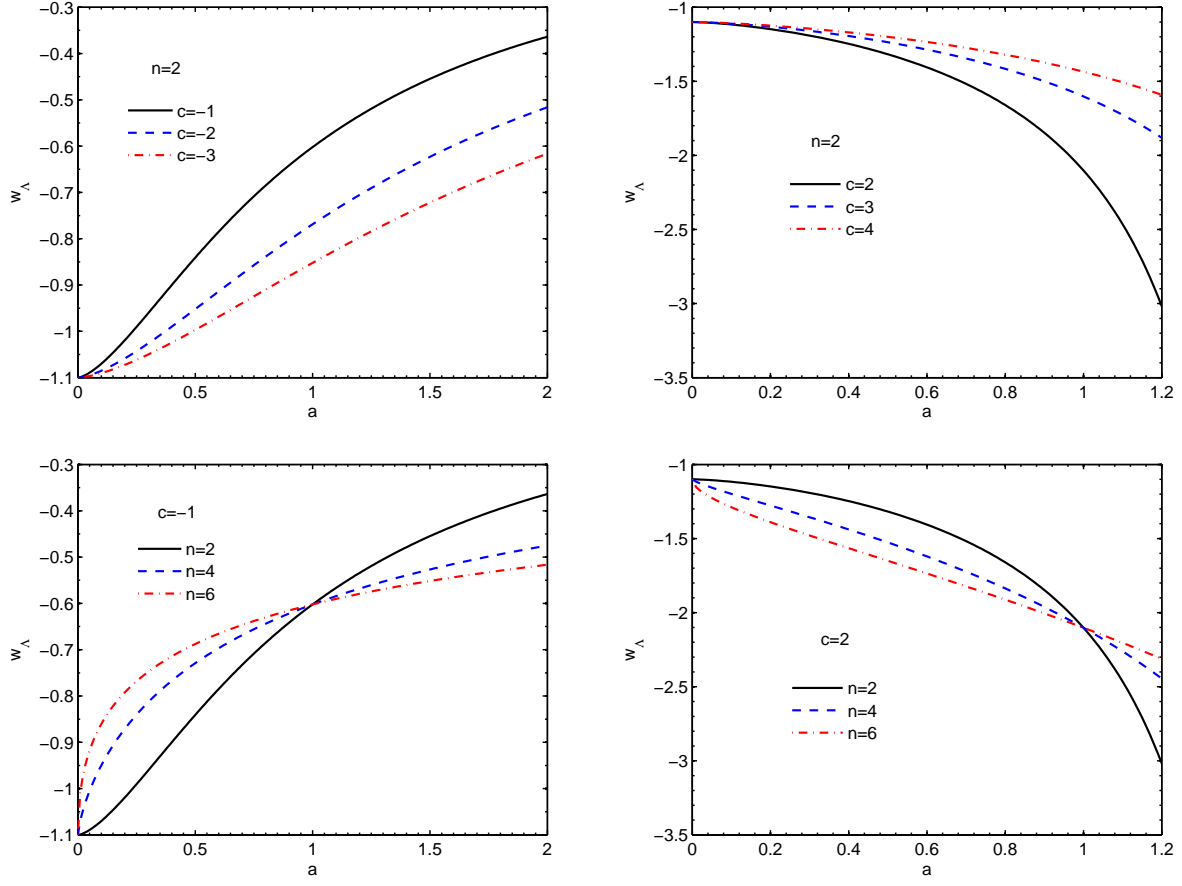


FIG. 1: The EoS parameter w_Λ of interacting polytropic gas model as a function of cosmic scale factor a for different model parameters c and n . The interaction parameter is chosen as $\alpha = 0.1$.

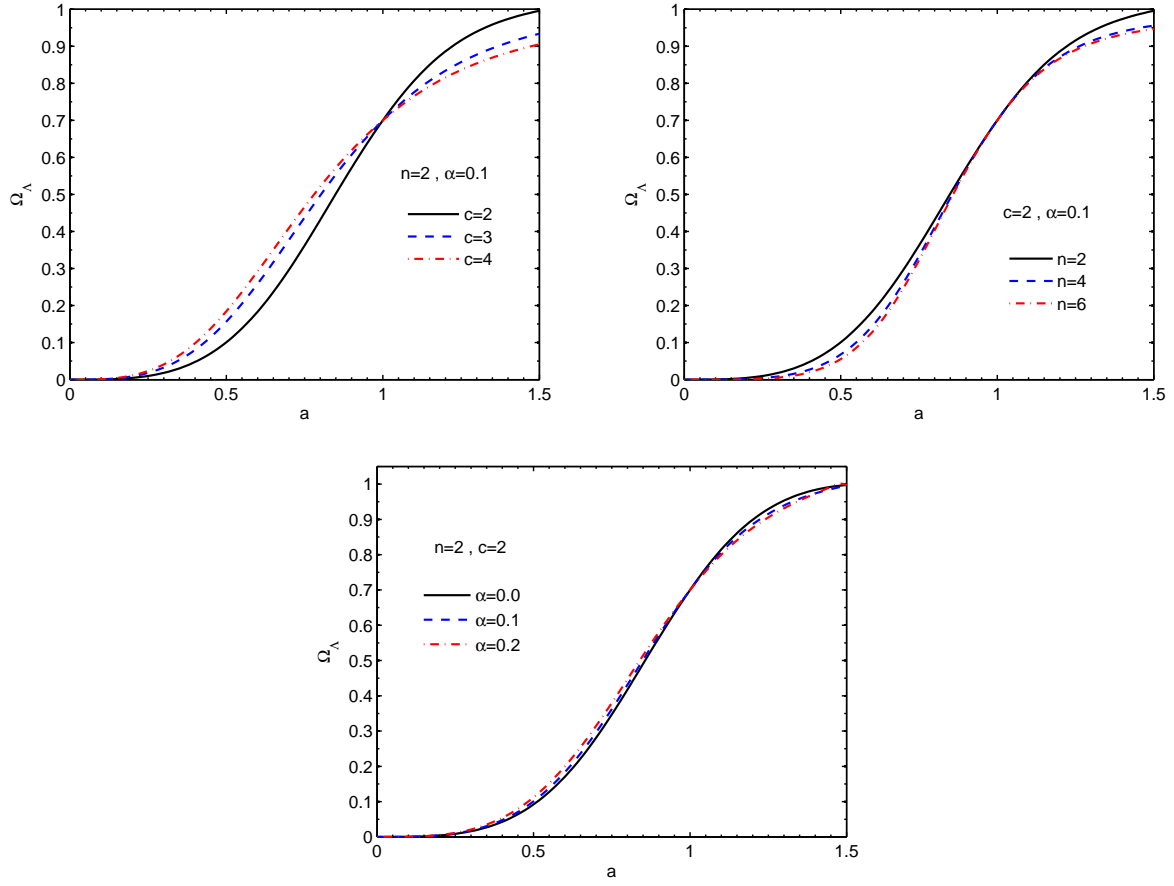


FIG. 2: The evolution of energy density Ω_Λ , in terms of cosmic scale factor a for interacting polytropic gas model. In upper left panel, the parameter n is fixed and the parameter c is varied. In upper right panel, we fix c and vary n . In lower panel, by fixing the parameters c and n , we vary the interaction parameter α .

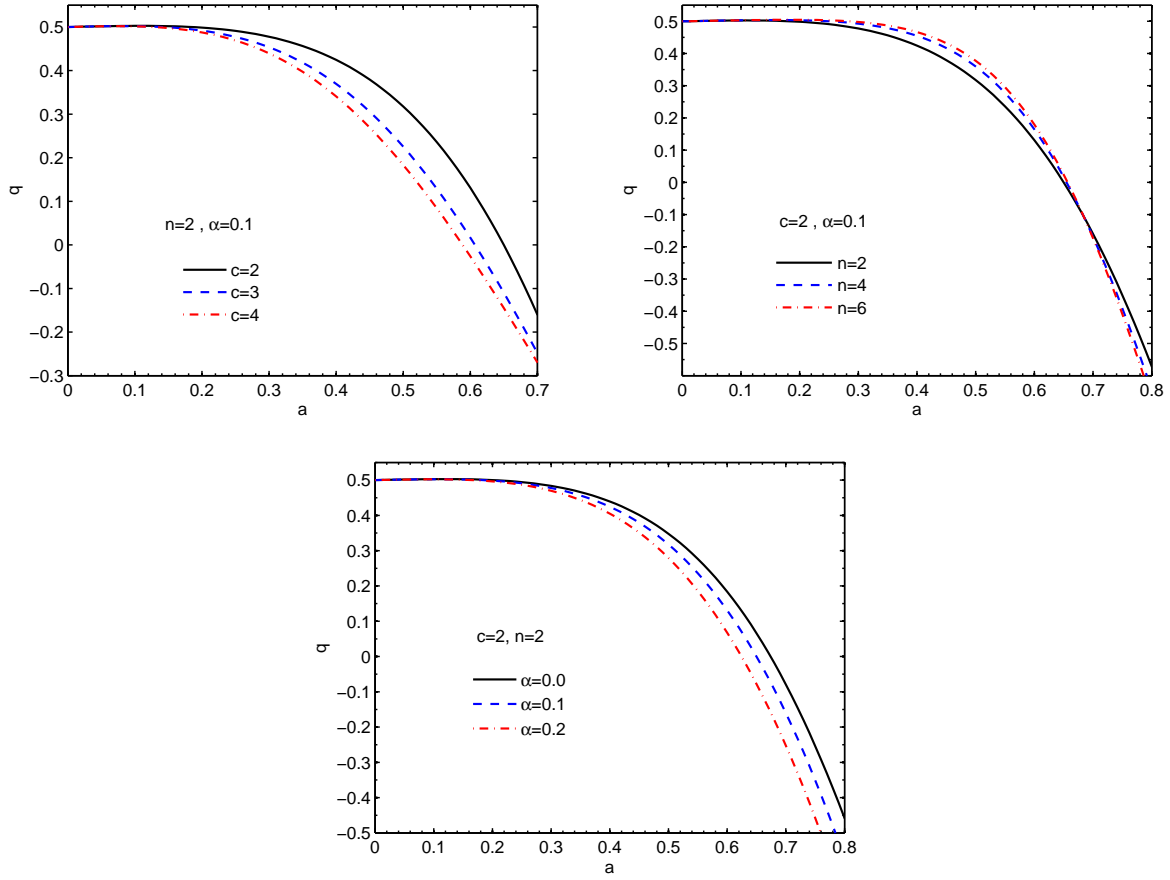


FIG. 3: The evolution of deceleration parameter q versus of scale factor a for interacting polytropic gas model. In upper left panel, the parameter n is fixed and the parameter c is varied. In upper right panel, we fix c and vary n . In lower panel, by fixing the parameters c and n , we vary the interaction parameter α .

-
- [1] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).
 - [2] C. L. Bennett et al., *Astrophys. J. Suppl.* **148**, 1 (2003).
 - [3] M. Tegmark et al., *Phys. Rev. D* **69**, 103501 (2004).
 - [4] S. W. Allen, et al., *Mon. Not. Roy. Astron. Soc.* **353**, 457 (2004).
 - [5] U. Alam, V. Sahni and A. A. Starobinsky, *JCAP* **0406** (2004) 008; D. Huterer and A. Cooray, *Phys. Rev. D* **71** (2005) 023506; Y.G. Gong, *Int. J. Mod. Phys. D* **14** (2005) 599; Y.G. Gong, *Class. Quantum Grav.* **22** (2005) 2121; Yun Wang and M. Tegmark, *Phys. Rev. D* **71** (2005)

- 103513; Yun-gui Gong and Yuan-Zhong Zhang, Phys. Rev. D **72** (2005) 043518.
- [6] C. Wetterich, Nucl. Phys. B **302**, 668 (1988);
B. Ratra, J. Peebles, Phys. Rev. D **37**, 321 (1988).
- [7] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002);
S. Nojiri, S.D. Odintsov, Phys. Lett. B **562**, 147 (2003);
S. Nojiri, S.D. Odintsov, Phys. Lett. B **565**, 1 (2003).
- [8] E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D **70**, 043539 (2004);
S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005);
A. Anisimov, E. Babichev, A. Vikman, J. Cosmol. Astropart. Phys. **06**, 006 (2005).
- [9] T. Chiba, T. Okabe, M. Yamaguchi, Phys. Rev. D **62**, 023511(2000);
C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000);
C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).
- [10] A. Sen, J. High Energy Phys. **04**, 048 (2002);
T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002);
T. Padmanabhan, T.R. Choudhury, Phys. Rev. D **66**, 081301 (2002).
- [11] M. Gasperini, F. Piazza, G. Veneziano, Phys. Rev. D **65**, 023508 (2002); N. Arkani-Hamed,
P. Creminelli, S. Mukohyama, M. Zaldarriaga, J. Cosmol. Astropart. Phys. **04**, 001 (2004); F.
Piazza, S. Tsujikawa, J. Cosmol. Astropart. Phys. **07**, 004 (2004).
- [12] A. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B **511**, 265 (2001); M. C. Bento, O.
Bertolami, A. A. Sen, Phys. Rev. D **66**, 043507 (2002);
- [13] M. R. Setare, Eur. Phys. J. C **52**, 689, 2007.
- [14] C. Deffayet, G. R. Dvali, G. Gabadaaze, Phys. Rev. D **65**, 044023 (2002); V. Sahni, Y.
Shtanov, J. Cosmol. Astropart. Phys. **0311**, 014 (2003).
- [15] P. Horava, D. Minic, Phys. Rev. Lett. **85**, 1610 (2000); P. Horava, D. Minic, Phys. Rev. Lett.
509, 138 (2001); S. Thomas, Phys. Rev. Lett. **89**, 081301 (2002); M. R. Setare, Phys. Lett.
B **644**, 99, 2007; M. R. Setare, Phys. Lett. B **654**, 1, 2007; M. R. Setare, Phys. Lett. B **642**,
1, 2006; M. R. Setare, Eur. Phys. J. C **50**, 991, 2007; M. R. Setare, Phys. Lett. B **648**, 329,
2007; M. R. Setare, Phys. Lett. B **653**, 116, 2007.
- [16] R.G. Cai, Phys. Lett. B **657**, (2007) 228; H. Wei, R.G. Cai, Phys. Lett. B **660**, 113 (2008).
- [17] G. t Hooft, gr-qc/9310026; L. Susskind, J. Math. Phys. **36**, 6377 (1995).
- [18] F. Karolyhazy, Nuovo.Cim. A **42** (1966) 390; F. Karolyhazy, A. Frenkel and B. Lukacs, in

- Physics as natural Philosophy edited by A. Shimony and H. Feschbach, MIT Press, Cambridge, MA, (1982); F. Karolyhazy, A. Frenkel and B. Lukacs, in Quantum Concepts in Space and Time edited by R. Penrose and C.J. Isham, Clarendon Press, Oxford, (1986).
- [19] O. Bertolami, F. Gil Pedro, M. Le Delliou, Phys. Lett. B **654**, 165 (2007).
 - [20] C. Feng, B. Wang, Y. Gong, R.K. Su, JCAP **0709**, 005 (2007).
 - [21] C.L. Bennett et al., Astrophys. J. Suppl. **148**, 1 (2003); D.N. Spergel, Astrophys. J. Suppl. **148**, 175 (2003); M. Tegmark et al., Phys. Rev. D **69**, 103501 (2004); U. Seljak, A. Slosar, P. McDonald, J. Cosmol. Astropart. Phys. **10**, 014 (2006); D.N. Spergel et al., Astrophys. J. Suppl. **170**, 377 (2007).
 - [22] U. Mukhopadhyay and S. Ray, Mod. Phys. Lett. A **23**, 3198,2008.
 - [23] K. Karami, A. Abdolmaleki, Astrophys. Space Sci.**330**, 133,2010.
 - [24] K. Karami, S. Ghaffari, J. Fehri, Eur. Phys. J. C, **64**, 85 (2009).
 - [25] S. Nojiri, S. D. Odintsov, S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005).
 - [26] H. Wei, R. G. Cai, Eur.Phys.J.C**59**, 105, 2009.
 - [27] K. Karami, A. Abdolmaleki, arXiv:1009.3587.
 - [28] J. Christensen-Dalsgard, *Lecture Notes on Stellar Structure and Evolution*, 6th edn. (Aarhus University Press, Aarhus, 2004).
 - [29] D. N. Spergel, et al., Satrophys. J. Suppl. **170**, 377 (2007).
 - [30] A. Sheykhi, Phys. Lett. B **680**, 113 (2009); H. Wei & R. G. Cai, Phys. Lett. B **660**, 113 (2008); L. Zhang, J. Cui, J. Zhang & X. Zhang, Int. J. Mod. Phys. D **19**, 21 (2010).
 - [31] L. P. Chimento, Phys. Rev. D **81**, 043525 (2010).
 - [32] L. Amendola, Phys. Rev. D **60** (1999) 043501;
 L. Amendola, Phys. Rev. D **62** (2000) 043511;
 L. Amendola and C. Quercellini, Phys. Rev. D **68** (2003) 023514;
 L. Amendola and D. Tocchini-Valentini, Phys. Rev. D **64** (2001) 043509 ;
 L. Amendola and D. T. Valentini, Phys. Rev. D **66** (2002) 043528.
 - [33] J. S. Bagla, H. K. Jassal, T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003), astro-ph/0212198
 Ying Shao, Yuan-Xing Gui and Wei Wang, Mod. Phys. Lett. A **22**, 1175-1182 (2007),
 gr-qc/0703112 Gianluca Calcagni and Andrew R. Liddle, Phys. Rev. D **74**, 043528,2006,
 astro-ph/0606003 Edmund J. Copeland, Mohammad R. Garousi, M. Sami and Shinji Tsu-
 jikawa, Phys. Rev. D **71**, 043003 (2005), hep-th/0411192

- [34] A. Sen, JHEP **0204**, 048 (2002); JHEP **0207**, 065 (2002); Mod. Phys. Lett. A **17**, 1797 (2002); arXiv: hep-th/0312153; A. Sen, JHEP **9910**, 008 (1999); E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras, S. Panda, JHEP **0005**, 009 (2000); J. Kluson, Phys. Rev. D **62**, 126003 (2000); D. Kutasov and V. Niarchos, Nucl. Phys. B **666**, 56, (2003).
- [35] A. Sen, Mod. Phys. Lett. A **17** (2002) 1797; N. D. Lambert, I. Sachs, Phys. Rev. D **67** (2003) 026005.
- [36] T. Chiba et al., Phys. Rev. D **62** (2000) 023511; C. Armendariz-Picon et al., Phys. Rev. Lett **85** (2000) 4438; C. Armendariz-Picon et al., Phys. Rev. Lett **63** (2001) 103510.
- [37] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004).